

# Applying Multi-sensor Information Fusion Technology to Problems of National Defense in the Complicated Social, Information, and Communication Environment

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## Abstract

In 2014, commercial flight MH17 was downed in Ukraine with 293 passengers aboard, US fighter Jets began striking ISIS terrorists in IRU, Japan and China confront each other over rights to an island in the South China Sea, and North Korea and Iran continue to threaten the world with nuclear weapon development. All these events indicate that we are still living in a world of conflict. There is a need to make the correct national defense decision under the complicated social, information and communication environment.

This paper proposes a best possible decision making model by comparing the Bayesian Fusion Model and the Dempster-Shafer Fusion Model. In complicated situations, decisions are not unique, both Bayesian and Dempster-Shafer can provide the final resolution.

In this comparison study, two examples are used to show the differences and similarities of the two fusion models. The first example is a real life problem in defense involving discrimination of friendly aircraft from enemy aircraft with radar, electronic warfare, and CNI sensors. The second example is from medicine. The two methods are used by a team of medical experts to positively discriminate between patients with a brain tumor, concussion, or meningitis.

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## Introduction

The main objective of this comparison study is to explore multi-sensor fusion technology and its applications. The multi-sensor Bayesian fusion model and the Dempster-Shafer fusion model are the most popular and widely applied fusion models. Both Bayesian fusion model and Dempster-Shafer fusion model have been widely applied in defense, medicine, and meteorology, for automatic classification and positive identification problems. In this comparison study, we try to find the mathematical expression for both the Bayesian fusion model and the Dempster-Shafer fusion model. We will also explain the differences and similarities between the Bayesian fusion model and the Dempster-Shafer fusion model.

In future international conflicts, social media such as smart phones with cameras, Facebook, and other internet resources will be part of the human intelligence. Advanced avionics systems, with multi-sensor fusion tracking, will be involved in beyond-viewing-range surveillance tracking targets of interest in space out to 120 miles. Remote sensing techniques through satellites, GPS, and Google maps will return clear images of any target from space as the future advanced reconnaissance.

In the multi-sensor, multiple target environment, there is a need for more advanced mathematical techniques like Bayesian fusion model and Dempster-Shafer fusion model to support the complicated intelligence, surveillance, and reconnaissance (ISR) mission. Future ISR missions, require logical decision making. The authors of this paper introduce a mathematical formula for Bayesian fusion model and derive a new formula to calculate probability for each target of interest from multiple sensor data via Dempster-Shafer fusion model.

In order to better understand the two multi-sensor fusion models, we focus on comparison of the Bayesian fusion model and the Dempster-Shafer fusion model. The Bayesian fusion model uses “probability” to expressed its decision, whereas Dempster-Shafer fusion model uses “mass” to express its decision. The two models are not numerically identical, but the models give the same conclusion for any given problem. The authors provide a simple mathematical expression to demonstrate the equivalency of Bayesian fusion model and Dempster-Shafer fusion model.

The Bayesian fusion model is based on the conditional probability model. It was developed by the British mathematician Thomas Bayes in the seventeen century. The Dempster-Shafer fusion model was developed by Arthur P. Dempster, a professor from Harvard University, and Glenn Shafer, a professor from Princeton University.

The Bayesian fusion model is a probability model whereas the Dempster-Shafer fusion model is a logical and mathematical model. The Bayesian fusion model requires a multivariate normal distribution assumption, whereas the Dempster-Shafer fusion model does not require a probability distribution assumption.

In the information age, as electronics and computer technology advance day-by-day, more sensor products are used, meaning more Information needs to be extracted and properly applied. This kind of problem is a great challenge for people in the field of command and control.

In this comparison study, we provide two examples. The first example from defense, applies the models to the problem of positively identifying multiple targets in a multi-sensor environment. The second example from medicine, applies the models to the emergency room “diagnosis problem”. In both these examples, we apply the Bayesian fusion model and the Dempster-Shafer fusion model to prove that both fusion models cause the same decision from the same data set or numerical problem.

Historically, Bayesian fusion model only works on data that meet the Normal Distribution assumption. Professors Dempster and Shafer tried to extended this probabilistic assumption. For this reason, Dempster-Shafer fusion model is known as the generalization of Bayesian fusion model.

## Bayesian Fusion Model

The fused target probability in the multi-sensor and multiple target environment can be expressed as [Hall, 1992]:

$$P\{ T_k / (S_1, S_2, S_3, \dots, S_n) \} = \{ P(S_1/T_k) \times P(S_2/T_k) \times P(S_3/T_k) \times \dots \times P(S_n/T_k) \} / \{ \sum_{i=1, n} [ P(S_1/T_i) \times P(S_2/T_i) \times P(S_3/T_i) \times \dots \times P(S_n/T_i) ] \}$$
 equation A [Hall, 1992]

Where:  $P\{ T_k / (S_1, S_2, S_3, \dots, S_n) \}$  is the probability of target  $T_k$  based on  $N$  sensors.

For the single sensor with multiple targets, the fused target probability reduces to the following:

$$P(T_k / S) = \{ P(S/T_k) \} / \{ \sum_{i=1, n} [ P(S/T_i) ] \}$$
 [Jeun, 1979]

This expression is the regular Bayesian probability model.

## Dempster-Shafer Fusion Model [Glenn Shafer, 1976]

The Dempster-Shafer theory was first developed by Professor Arthur P. Dempster, at Harvard University in the 1960’s. It was later extended by Professor Glenn Shafer at Princeton University. Later on, the new theory was known as the theory of evidence. The Dempster-Shafer theory, in general is a collection of evidence from multiple sources used to form logical decisions to interpret uncertainty.

The most important part of Dempster-Shafer theory is Dempster’s rule of combinations. A combination (called a **joint mass**) is calculated from two sets of masses  $m_1$  and  $m_2$  in the following manner.

The two dimensional matrix of probability mass for two sensors (with mass  $m_1$  and  $m_2$ ), and three targets (A, B and C), can be constructed as following:

Table 1 Probability Mass for 2 sensors and 3 targets

|    |   | m2                     |                        |                        |
|----|---|------------------------|------------------------|------------------------|
|    |   | A                      | B                      | C                      |
| m1 | A | $m_1(A) \times m_2(A)$ | $m_1(A) \times m_2(B)$ | $m_1(A) \times m_2(C)$ |
|    | B | $m_1(B) \times m_2(A)$ | $m_1(B) \times m_2(B)$ | $m_1(B) \times m_2(C)$ |

|  |   |                        |                        |                        |
|--|---|------------------------|------------------------|------------------------|
|  | C | $m_1(C) \times m_2(A)$ | $m_1(C) \times m_2(B)$ | $m_1(C) \times m_2(C)$ |
|--|---|------------------------|------------------------|------------------------|

The probability mass for targets A, B, and C in the two sensor environment can be defined as following:

$$\text{Pr-m}(A) = \{ 1.0 / (1.0 - K) \} \{ m_1(A) * m_2(A) \}$$

$$\text{Pr-m}(B) = \{ 1.0 / (1.0 - K) \} \{ m_1(B) * m_2(B) \}$$

$$\text{Pr-m}(C) = \{ 1.0 / (1.0 - K) \} \{ m_1(C) * m_2(C) \}$$

where  $K$  = the measure of conflict between the two sensors

$$= \{ m_1(A) * m_2(B) + m_1(A) * m_2(C) \} \\ + \{ m_1(B) * m_2(A) + m_1(B) * m_2(C) \} \\ + \{ m_1(C) * m_2(A) + m_1(C) * m_2(B) \}$$

and  $(1.0 - K)$  = the normalization factor

The two dimensional matrix of probability mass for two sensors (with mass  $m_1$  and  $m_2$ ) and multiple targets ( $T_1, T_2, T_3, \dots T_n$ ) can be constructed as following:

**Table 2 Probability Mass for 2 sensors and multiple targets**

|    |     | m2                         |                            |                            |     |                            |
|----|-----|----------------------------|----------------------------|----------------------------|-----|----------------------------|
|    |     | T1                         | T2                         | T3                         | ... | Tn                         |
| m1 | T1  | $m_1(T_1) \times m_2(T_1)$ | $m_1(T_1) \times m_2(T_2)$ | $m_1(T_1) \times m_2(T_3)$ |     | $m_1(T_1) \times m_2(T_n)$ |
|    | T2  | $m_1(T_2) \times m_2(T_1)$ | $m_1(T_2) \times m_2(T_2)$ | $m_1(T_2) \times m_2(T_3)$ |     | $m_1(T_2) \times m_2(T_n)$ |
|    | T3  | $m_1(T_3) \times m_2(T_1)$ | $m_1(T_3) \times m_2(T_2)$ | $m_1(T_3) \times m_2(T_3)$ |     | $m_1(T_3) \times m_2(T_n)$ |
|    | ... |                            |                            |                            |     |                            |
|    | Tn  | $m_1(T_n) \times m_2(T_1)$ | $m_1(T_n) \times m_2(T_2)$ | $m_1(T_n) \times m_2(T_3)$ |     | $m_1(T_n) \times m_2(T_n)$ |

The probability mass for multiple targets  $T_1, T_2, T_3, \dots T_n$  in the two sensor environment can be defined as following:

$$\text{Pr-m}(T_1) = \{ 1.0 / (1.0 - K) \} * \{ m_1(T_1) * m_2(T_1) \}$$

$$\text{Pr-m}(T_2) = \{ 1.0 / (1.0 - K) \} * \{ m_1(T_2) * m_2(T_2) \}$$

$$\text{Pr-m}(T_3) = \{ 1.0 / (1.0 - K) \} * \{ m_1(T_3) * m_2(T_3) \}$$

-----

$$\text{Pr-m}(T_n) = \{ 1.0 / (1.0 - K) \} * \{ m_1(T_n) * m_2(T_n) \}$$

where  $K$  = the measure of conflict between the two sensors



The Bayesian fusion model can solve multi-sensor and multiple target problems, with just one single substitution to reach a solution. The Dempster-Shafer fusion model requires multiple steps and repeated procedures to reach a final solution when solving multi-sensor and multiple target problems. Refer to example #1 in this comparison study to see the solution of a three sensor and four target problem. The Dempster-Shafer fusion model requires two steps to reach the final solution.

## Application in Defense (Example #1)



Figure 1 Target1 (F22 fighter aircraft), Target2 (P3 ASW aircraft), Target3 (Malaysian MH370)

Detection, tracking, and positive Identification of aircraft in the multi-sensor environment is a very difficult and challenging defense task. Each individual sensor, such as radar, electronic warfare (EW) and communication, navigation, identification (CNI) may not give identical information. In that case, a decision conflict may occur. Conflict resolution is needed. Usually multi-sensor information fusion technology is used to resolve such conflicts and provide a decision. For example, in a three sensor (radar, EW, and CNI) and three target (T1—F22, T2—P3, and T3—MH370) problem, probabilities of detection associated to each target are indicated in the following table. The fourth target T4, the undetermined aircraft, is used to account for uncertainty in the sensors.

Table 3 Example Probability of Detection Data

|            | T1 - F22 | T2 - P3 | T3 – MH370 | T4 – undetermined aircraft |
|------------|----------|---------|------------|----------------------------|
| Radar (S1) | 0.80     | 0.05    | 0.05       | 0.10                       |
| EW (S2)    | 0.30     | 0.60    | 0.05       | 0.05                       |
| CNI (S3)   | 0.20     | 0.70    | 0.05       | 0.05                       |

The above table strongly indicates that the three sensors, Radar, EW and CNI, do not have an identical decision on what target has been detected.

### Solution by Bayesian Fusion Model

Now applying the Bayesian fusion model to the data set of example #1 gives the equation for four targets and three sensors as following:

$$Pr( Tk/S1,S2,S3) = \{ Pr(S1/Tk)*Pr(S2/Tk)*Pr(S3/Tk)\} /$$

$$\{ \sum \Pr( S1/Ti) * \Pr(S2/Ti) * \Pr(S3/Ti) \} \quad i=1,2,3 \quad \text{equation < A >}$$

From the sensor information given in example #1, we have:

$$\Pr( S1/T1) = 0.80$$

$$\Pr( S2/T1) = 0.30$$

$$\Pr( S3/T1) = 0.20$$

$$\Pr( S1/T2) = 0.05$$

$$\Pr( S2/T2) = 0.60$$

$$\Pr(S3/T2) = 0.70$$

$$\Pr(S1/T3) = 0.05$$

$$\Pr(S2/T3) = 0.05$$

$$\Pr(S3/T3)=0.05$$

$$\Pr(S1/T4)= 0.10$$

$$\Pr(S2/T4)=0.05$$

$$\Pr(S3/T4)=0.05 \quad \text{equation < B >}$$

By substituting equation < B > into equation <A>, we have:

$$\Pr( T1 / S1, S2, S3 ) = \{ (0.80 * 0.30 * 0.20) \} /$$

$$\{ (0.80 * 0.30 * 0.20) + (0.05 * 0.60 * 0.70) + (0.05 * 0.05 * 0.05) + (0.10 * 0.05 * 0.05) \}$$

$$= 0.6919 \quad \text{or} \quad 69.19 \%$$

Therefore, the fused probability for target T1 (F22 fighter jet) is

$\Pr(T1/S1,S2,S3) = 69.19 \%$  in the three sensor (Rader, EW and CNI) environment.

Similarly, for target 2 we get:

$$\Pr( T2/S1, S2, S3) = 0.3027 \quad \text{or} \quad 30.27 \%$$

and for target 3 we get:

$$\Pr( T3/S1,S2,S3 ) = 0.0018 \quad \text{or} \quad 0.18\%$$

and for the undetermined target, target 4, we get:



$$\Pr(T4/S1,S2,S3) = 0.0036 \text{ or } 0.36\%$$

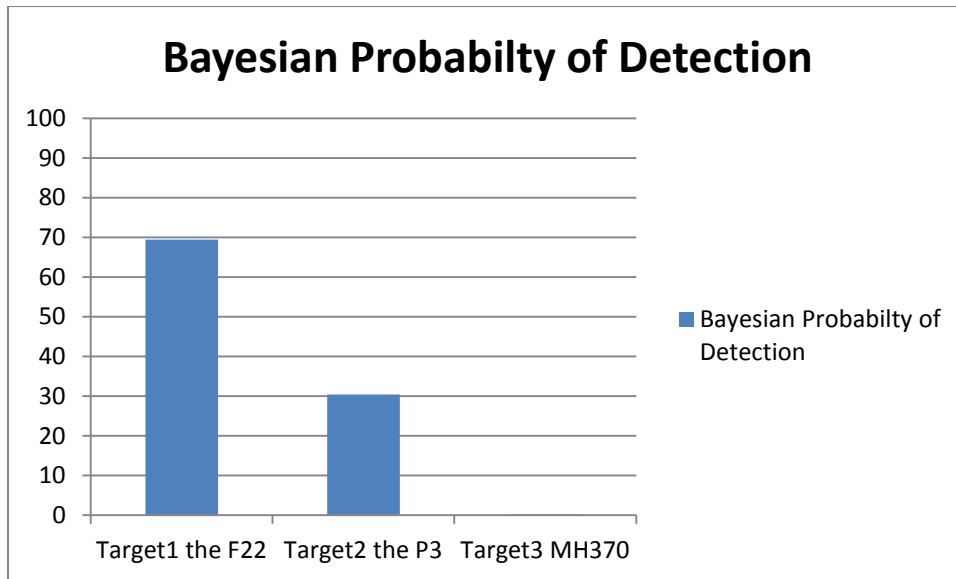
Therefore, T1 (the F22) is detected with probability of 69.19%,

T2 (the P3) is detected with probability of 30.27%, and

T3 (MH370) is detected with probability of 0.18%, and

T4( other aircraft) is detected with probability of 0.36%

in the three sensor (Radar, EW, CNI) environment.



### Solution by Dempster-Shafer Fusion Model

Now, we will apply the Dempster-Shafer fusion model to example #1. Since it is a three-sensor and four-target problem, it requires two steps for a complete solution. Step one - construct a two dimensional matrix of a probability mass for Radar and EW and find the probability mass for each target. Step two - using the results from step one, construct a new two dimensional matrix of probability mass, and find the probability mass for all targets from the remaining CNI sensor.

Step 1:

Table 4 Probability Mass of Detection of 4 targets from 2 sensors

|        |          | S1 (radar) |          |          |         |
|--------|----------|------------|----------|----------|---------|
|        |          | T1(0.8)    | T2(0.05) | T3(0.05) | T4(0.1) |
| S2(EW) | T1(0.3)  | 0.24       | 0.015    | 0.015    | 0.03    |
|        | T2(0.6)  | 0.48       | 0.030    | 0.030    | 0.06    |
|        | T3(0.05) | 0.04       | 0.0025   | 0.0025   | 0.005   |
|        | T4(0.05) | 0.04       | 0.0025   | 0.0025   | 0.005   |

From the above two dimensional matrix of probability mass, we have:

$$K = 0.015 + 0.015 + 0.03 + 0.48 + 0.03 + 0.06 + 0.04 + 0.0025 + 0.005 + 0.04 + 0.0025 + 0.0025 = 0.7225 \text{ as the conflict.}$$

$$1.0 - K = 0.2775 \text{ as the normalization factor}$$

and the required probability mass for target 1, target 2, target 3, and target 4 can be obtained as:

$$\text{Pr}_m(\text{T1}) = (1.0/0.2775) * 0.240 = 0.8649 \text{ or } 86.49 \%$$

$$\text{Pr}_m(\text{T2}) = (1.0/0.2775) * 0.030 = 0.1081 \text{ or } 10.81 \%$$

$$\text{Pr}_m(\text{T3}) = (1.0/0.2775) * 0.0025 = 0.009 \text{ or } 0.9 \%$$

$$\text{Pr}_m(\text{T4}) = (1.0/0.2775) * 0.005 = 0.018 \text{ or } 1.8\%$$

$\text{Pr}_m(\text{T1})$  is the probability mass for Target T1, the F22

$\text{Pr}_m(\text{T2})$  is the probability mass for Target T2, the P3

$\text{Pr}_m(\text{T3})$  is the probability mass for Target T3, the airliner

$\text{Pr}_m(\text{T4})$  is the probability mass for the undetermined aircraft type T4

Step #2: now use the results obtained from Radar and EW, to construct probability mass matrix for Sensor CNI:

**Table 5 Probability Mass after adding 3rd sensor**

|          |          | S1 & S2 (Radar & EW) |            |           |           |
|----------|----------|----------------------|------------|-----------|-----------|
|          |          | T1(0.8649)           | T2(0.1081) | T3(0.009) | T4(0.018) |
| S3 (CNI) | T1(0.2)  | 0.1730               | 0.0216     | 0.0018    | 0.0036    |
|          | T2(0.7)  | 0.6054               | 0.0757     | 0.0063    | 0.0126    |
|          | T3(0.05) | 0.0432               | 0.0054     | 0.0005    | 0.0009    |
|          | T4(0.05) | 0.0432               | 0.0054     | 0.0005    | 0.0009    |

From the above probability mass matrix now including sensor CNI, we find

$$K = 0.0216 + 0.0018 + 0.0036 + 0.6054 + 0.0063 + 0.0126 + 0.0432 + 0.0054 + 0.0009 + 0.0432 + 0.0054 + 0.0005 = 0.7499$$

$$1.0 - k = 0.2501$$

where K is the conflict and normalization factor for targets T1, T2, T3, and T4 in the three sensor S1, S2, and S3 environment.

$$\text{Pr}_m(\text{T1}) = (1/0.2501) * 0.1739 = 0.6917 \text{ or } 69.17\%$$

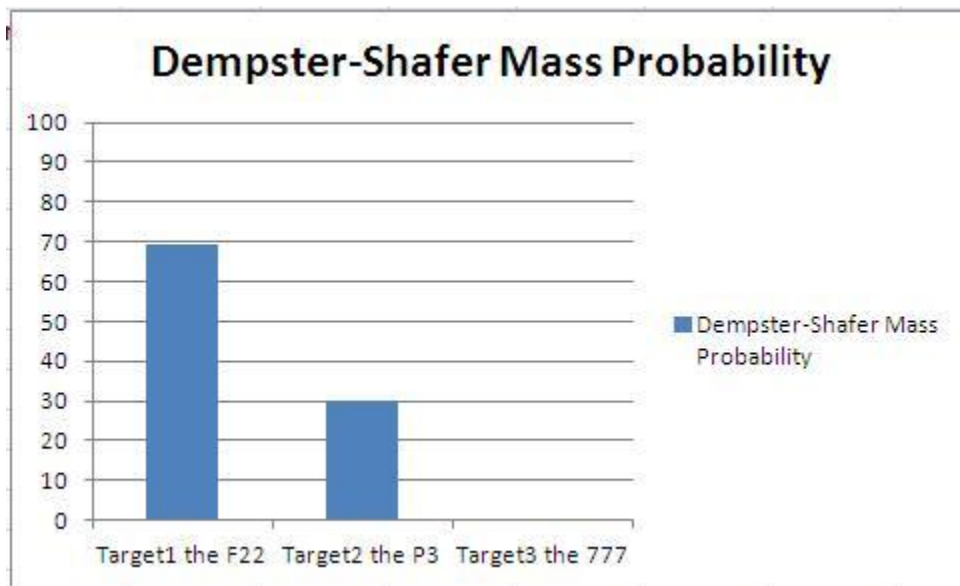
$$\text{Pr}_m(\text{T2}) = (1/0.2501) * 0.0757 = 0.3027 \text{ or } 30.27\%$$

$$\text{Pr}_m(\text{T3}) = (1/0.2501) * 0.0005 = 0.002 \text{ or } 0.2\%$$

$$\text{Pr}_m(\text{T4}) = (1/0.2501) * 0.0009 = 0.0036 \text{ or } 0.36\%$$

Therefore, the probability mass for Target T1 (the F22) is 69.17 %, the probability mass for Target T2 (the P3) is 30.27%, the probability mass for Target T3 (MH370) is 0.2%, and the probability mass of the undetermined target is 0.36% in the three sensor environment.

The results provide the same conclusion as the solution provided by the Bayesian fusion model.



## Application in Medicine (Example #2)

In example #2, a patient is diagnosed by two neurologists - doctor A and doctor B. The doctors express their diagnosis as a probability of three possible diseases:

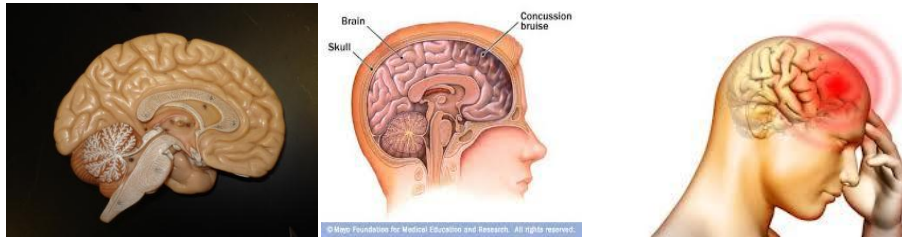
Table 6 Probability of Disease

|          | T1 (brain tumor) | T2 (concussion) | T3 (meningitis) |
|----------|------------------|-----------------|-----------------|
| Doctor A | 0.75             | 0.20            | 0.05            |
| Doctor B | 0.80             | 0.12            | 0.08            |

The purpose of this example is to demonstrate how to discriminate between brain tumor, concussion, and meningitis when two brain doctors are involved. Just looking at the data, one can conclude that the patient is most likely suffering from a brain tumor. Now we will apply the Bayesian fusion model and

Dempster-Shafer fusion model to the data of the two neurologists. We will compare the answers obtained by the two models and check whether the results agree or disagree.

Figure 2 Brain Tumor, Brain Concussion, Meningitis



First, we determine that example #2 is a two sensor and three target problem. S1 denotes doctor A and S2 denotes doctor B. Target T1 denotes the perceived probability of the disease being a brain tumor, target T2 denotes concussion, and target T3 denotes Meningitis.

### Solution by the Bayesian Fusion Model:

To apply the Bayesian fusion Model, we obtain a formula that can provide a method to estimate fused probabilities for T1, T2, and T3 from sensors S1, and S2 as following:

$$\Pr(T1/S1,S2) = \Pr(S1/T1) * \Pr(S2/T1) /$$

$$\{ [ \Pr(S1/T1) * \Pr(S2/T1) ]$$

$$+ [ \Pr(S1/T2) * \Pr(S2/T2) ]$$

$$+ [ \Pr(S1/T3) * \Pr(S2/T3) ] \}$$

$$\Pr(T2/S1,S2) = \Pr(S1/T2) * \Pr(S2/T2) /$$

$$\{ [ \Pr(S1/T1) * \Pr(S2/T1) ]$$

$$+ [ \Pr(S1/T2) * \Pr(S2/T2) ]$$

$$+ [ \Pr(S1/T3) * \Pr(S2/T3) ] \}$$

$$\Pr(T3/S1,S2) = \Pr(S1/T3) * \Pr(S2/T3) /$$

$$\{ [ \Pr(S1/T1) * \Pr(S2/T1) ]$$

$$+ [ \Pr(S1/T2) * \Pr(S2/T2) ]$$

$$+ [ \Pr(S1/T3) * \Pr(S2/T3) ] \}$$
 equation < A >

and from the example #2 given information, we have:

$$\Pr(S1/T1) = 0.75$$

$$\Pr(S1/T2) = 0.20$$

$$\Pr(S1/T3) = 0.05$$

$$\Pr(S2/T1) = 0.80$$

$$\Pr(S2/T2) = 0.12$$

$$\Pr(S2/T3) = 0.08 \quad \text{equation < B >}$$

Now, substituting equation < B > into equation < A >, we have:

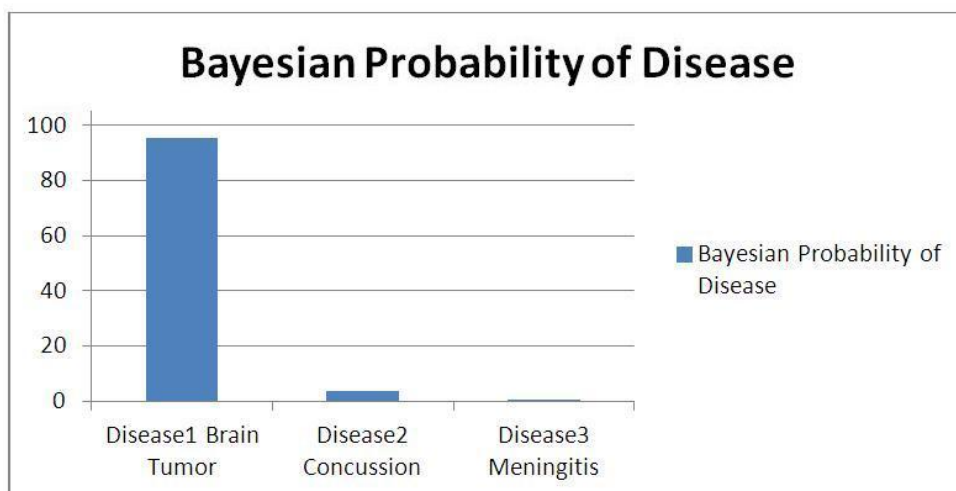
$$\begin{aligned} \Pr(T1/S1,S2) &= (0.75) * (0.80) / \\ &\quad \{ [(0.75)*(0.80)] \\ &\quad + [(0.20)*(0.12)] \\ &\quad + [(0.05)*(0.08)] \} \\ &= 0.9554 \end{aligned}$$

That is, the doctors are 95.54 % certain that the patient is suffering from a brain tumor.

Similarly, we have:

$$\Pr(T2/S1,S2) = 0.024 / 0.628 = 0.0382 \quad \text{or} \quad 3.82\% \quad \text{probability of concussion}$$

$$\Pr(T3/S1,S2) = 0.004 / 0.628 = 0.0063 \quad \text{or} \quad 0.6\% \quad \text{probability of meningitis}$$



One can conclude that both Doctors A and B will similarly conclude the patient is suffering from a brain tumor with 95.54% certainty, suffering from a concussion with 3.82% certainty, and suffering from meningitis with 0.6% certainty.

### Solution by Dempster-Shafer Fusion Model:

Now, build a two dimensional matrix of sensors and targets to estimate the required probability mass for example #2 as following:

Table 7 Probability Mass

|               |          | S1 (doctor A) |          |          |
|---------------|----------|---------------|----------|----------|
|               |          | T1(0.75)      | T2(0.20) | T3(0.05) |
| S2 (doctor B) | T1(0.80) | 0.60          | 0.16     | 0.04     |
|               | T2(0.12) | 0.09          | 0.024    | 0.006    |
|               | T3(0.08) | 0.06          | 0.016    | 0.004    |

From the above two dimensional matrix of probability mass, we have:

$K = 0.16 + 0.04 + 0.09 + 0.006 + 0.06 + 0.016 = 0.372$  as the measure of conflict.

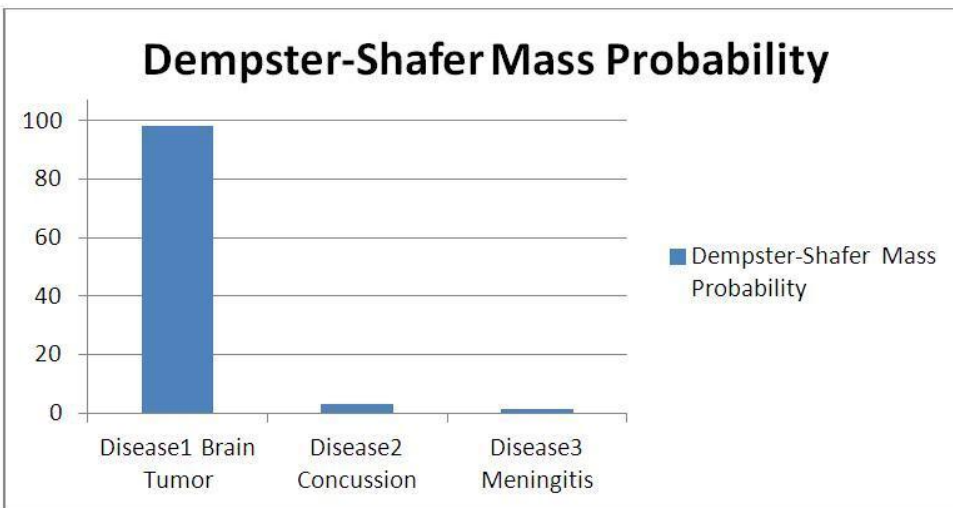
$1.0 - K = 0.628$  as the normalization factor

and the required probability mass for target 1, target 2, and target 3 are obtained as follows:

$Pr\_m(T1) = 0.60 / 0.628 = 0.9554$  Or 95.54 % probability of brain tumor

$Pr\_m(T2) = 0.02 / 0.628 = 0.0382$  or 3.82% probability of concussion

$Pr\_m(T3) = 0.01 / 0.628 = 0.006$  or 0.6% probability of meningitis



One can conclude that both doctors A and B agree that the patient is suffering from a brain tumor with 95.54 % certainty, suffering from a concussion with 3.82% certainty, and suffering from meningitis with 0.6% certainty.

## Conclusion

Applying the Bayesian fusion model and Dempster-Shafer fusion model to example #1 and example #2, both cause the same conclusion.

We can demonstrate mathematically that the Bayesian fusion model probability is a function of the Dempster-Shafer fusion model probability mass and measurement of confusion, K.

$$\Pr(T) = f\{ \Pr_m(T), K\} \quad \text{proof upon request}$$

Both Bayesian fusion model and Dempster-Shafer fusion model can solve multi-sensor and multiple target problems from the field of defense, medical, and other fields.

The Bayesian fusion model is a conditional probability model which requires multivariate normal distribution assumptions whereas the Dempster-Shafer fusion model is a mathematical and logical model which does not require multivariate normal distribution assumptions.

Both Bayesian fusion model and Dempster-Shafer fusion model have been used to verify numerical multi-sensor and multiple target problems with examples published in well known academic journals. [Dempster, 1968] and [Sandia.gov, 2002]

When compared to the Dempster-Shafer fusion model, the Bayesian fusion model is easier to use in solving multi-sensor and multiple target problems. The Bayesian fusion model requires simple substitution. Whereas the Dempster-Shafer fusion model requires multiple steps to reach a solution in the multi-sensor and multiple target environment [Example #1 and Example #2].

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