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## Frontiers of C2

Topic 8: Methodological Development, Experimentation, Analysis, Assessment and Metrics

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### *Integrating analysis of Planning and Execution in headquarters models*

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# Integrating analysis of Planning and Execution in headquarters models

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## Abstract

The occasional dysfunctionality between the J5 (Plans) and J3 (Operation Execution) branches of a military headquarters are well known – planners work to longer time-frames and standardised processes and thus often have a mismatched engagement with time-poor operators who work reactively to developing situations in theatre each time doing things a different way from the last. Ideally, the tight battle-rhythm of operators should feed into the longer cycles of the planners, and vice-versa. These effects flow-through to the other sections across a Joint Staff branches that interact with these 'two-hemispheres' of the headquarters. Almost as challenging is being able to analyse this disconnect in a single model of a headquarters. Apart from time-scales, the difference in level of irreducible complexity of tasks between activity in the two branches calls for quite different modelling methodologies; common representation thus needs higher levels of abstractions from traditional task decomposition approaches. In this paper we apply to this problem a recent advance in the study of synchronisation on networks, namely representations of *nested frequency distributions*. This builds on earlier work by the author establishing the famous Kuramoto model of synchronisation of oscillators on networks as appropriate for Command and Control modelling. I will apply the nested frequency form of the model to a typical Joint Staff structure and explore how this allows identification of overloaded staff areas and friction points in the structure.

## Introduction

Planning and Execution may be called the functions of the two *hemispheres* of a headquarters – be it military, commercial or scientific in its core business. To be specific, I mean 'deliberate planning' by the former as, when a plan suffers first contact with the adversary, 'execution' is essentially planning on the fly. Seen in this way, planning is a rational activity played out over – often – an extensive period of time. The available time and the scope for rationality allow for a strong formalisation of the stages of planning – a *planning process* (Mintzberg 1994). Because contact with the environment remains in the future (when the plan is executed), the situation is changing according to its intrinsic dynamic. Execution is quite different. Contact with the environment is copious so that the environment coevolves with the operators. The contingencies of this evolution defies pre-formulated schedules. To use another analogy from cognitive psychology (Morewedge and Kahneman 2010; Kahneman 2011): organisational planning is very much System 2 in nature, also called "slow thinking", while execution is System 1, or "fast thinking". These classifications emphasise the deliberative (system 2, or slow) versus the intuitive (system 1, or fast) processes of the individual human mind whose locus in the brain may be physically identified; hence my labelling Planning and Execution as two hemispheres of a headquarters, the *brain of an organisation* (Wilkinson 1890). That such a relationship exists is no surprise as the development of headquarters through the 19th century may be understood as the *unpacking of the mind* of the Commander or Chief Executive, and the *division of their labour*, as warfare and industry assumed continental and trans-Atlantic scales. This paper addresses an approach to model both types of thinking in an organisation – in a manner that permits the identification of trade-offs between the two 'hemispheres'.

A standard approach for modelling the conduct of work in a headquarters, indeed in many types of administrative and commercial organisations, is `process' or `workflow' modelling (Aguilar-Saven 2004). Here, the activities of individuals or teams are decomposed into discrete tasks of finite duration and (personnel or physical) resources, which are then arranged sequentially or in parallel. The visual representation or static analysis of these provides much insight into critical paths and bottlenecks; simulation through stochastic discrete event methods deepens this understanding further. In the past, I have contributed to such applications to headquarters planning, where the key steps of an operational planning process (Guitouni 2006) are represented in a linear waterfall approach: an event triggers Planning, with the sequence of Mission Analysis, Course of Action Development and Analysis, and Decision, where the model ends; sub-activities within these models often overlap in time, and multiple runs with stochastic variables for time and resources test the robustness of the proposed process and its resourcing (Kalloniatis and Wong 2007); see also (Grant et al 2008).

Execution of military operations is often analysed at the tactical level using agent based simulation (Dekker 2006; Manso 2012) where local agents may draw upon bottom up information about environment – often where other agents also represent adversaries or other actors in the battle-space – and make local decisions based on information observed directly, or based on information exchange with other agents according to the means available for communication. In my own work, I have focused on the cyclic nature of a headquarters 24 hour battle-rhythm with tasks flowing into, and out of, the Commander's daily brief, overlaid with stochasticity, to model the work of operations staff in a headquarters (Kalloniatis, Macleod and La 2009).

However, in both agent-based and my stochastic-cyclic approaches, the 'planning architecture' tends to be very simplistic, incomparable to the detail in process models of a planning process. Fundamentally, the problem in representing both planning and operations staff in a single model is because of the mismatch in levels of fidelity in the representation of the activity of the two staffs.

What is fundamental to both hemispheres of the headquarters is the intrinsic role of cycles. Planning cycles, for all the linearity invoked above, are repeated. Execution is managed in cycles balancing the needs of human physical existence (food, sleep) and maintaining engagement with the developing situation; battle-rhythm (Kamena 1999) will usually involve situation updates in the morning, and often also in the evening, then some rapid deliberation, and then allocation of tasking, waiting for things to develop and then repeat. I have long argued that this cyclic nature gives the elementary building block for modelling headquarters. In both cases, the cycle is a simplified or elaborated form of Boyd's OODA loop (Osinga 2013; Hasik 2013), or the basic Perception-Action cycle of Neisser (1976), or the feedback model of Endsley (2006), within which the three levels of Situation Awareness (SA) are usually the focus.

It is this absence of this role of cognitive cycles in other approaches to headquarters dynamical[1] modelling that distinguishes them from the approach taken here. One of the earliest such models is due to Coyle (1987) who exploited the dynamics of fluid flows within a Systems Dynamics approach, which has been generalised into the formalism of queueing theory by Levchuck et al. (2002). Thus information is abstracted here and flows as either a continuous or discrete entity. A different abstraction, that of OrgAhead (Carley and Svoboda 1996), sees the task of an organisational entity as processing bit-strings – converting to a "1" or "0" a string of bits flowing from the external environment or another entity in the organisation. Such an approach may be seen to be inspired by

[1] Examples of static – or even time-stamped – analysis of headquarters are traditional Social Network Analyses, such as (Jarvis 2005), or other weighted 'system analysis' network approaches (Lees and Bowden 2007).

the classical model of Statistical Physics, the Ising (1925) model of interacting spins on a lattice in either up or down states. More explicit use of the property of domain formation of this model for magnetisation as a paradigm for collaboration in C2 is by Song, Zhang and Qian (2013), who are partly inspired by the modelling of alliance formation by Axelrod and Bennett (1993). A related statistical physics inspired approach employs spin glasses, but models the hardware end of sensor networks (Iyengar and Brooks 2012) or more opinion spread in public or international contexts (Galam 2002). Finally, and returning to the modelling of information sharing, is the approach of Perry and Moffat (2004), which applies Shannon's Information Entropy to capture the role of uncertainty as situation awareness builds through a C2 structure. But here the model is built around quite specific headquarters processes, such as for Rapid Planning. Similar, and equally inspiring, applications of ideas from statistical physics are applied by Moffat (2003) to tactical information for the control of enemy forces in a battle-space. All models in science necessarily involve an abstraction of some real world phenomenon, but none of these representations of C2 capture the cyclic nature of human cognition and the range of tasks required at the operational level. What these models do provide is an instantiation in C2 of the phenomenon of *criticality* or *phase transitions*, where coherent structures or behaviour are spontaneously generated through bottom-up interactions[2], often with a small change of a variable such as `temperature'. The model I pursue here may also such behaviours; indeed it was formulated expressly to capture such possible phenomena. This is not to suggest that the approach I take in this work is intended to replace alternatives. Rather, I seek to complement them because the multi-fold nature of C2 requires diverse computational social models that are challenging to validate (Turnley and Perls 2008) in their own right but are best utilised in a cross-validation approach (Schreiber 2002; Bharathy and Silverman 2010).

As I have argued in the past (Kalloniatis 2008), the Kuramoto model (Kuramoto 1984) of synchronisation of phase oscillators on a network[3] provides the basic elements of the cycle, structure and heterogeneity in this lens on a headquarters; further stochasticity may be introduced to inject human intrinsic or environmental extrinsic heterogeneity (Kalloniatis and Zuparic 2014). For the first application of the Kuramoto model to C2 see, however, Dekker (2007). To date, my applications of this model (Kalloniatis 2012; Kalloniatis and Zuparic 2014; Kalloniatis 2016) have represented agents working within a common temporal frame. Recent developments in the physics literature (Terada and Aoyagi 2016) have shown how nested cycles may be represented by articulating an interaction that permits each set of agents, fast and slow, to self-synchronise to their temporal scale, while allowing interchange between the fast and slow processes. This provides a useful framework for uniting Planners and Operators, as execution is a fast activity nested within the slower deliberate planning process when the two are required to overlap[4]. Nevertheless, given the heterogeneous connectivity through the whole organisation – at some point some planners and operators must exchange information – the stresses of the two tempos can be represented and

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[2] In some places this phenomenon is called emergence but mathematical definitions of this remain a challenge (Bar-Yam 2004) so I will avoid its use here.

[3] For a recent thorough review of the literature on the Kuramoto model and its applications to diverse systems, including socio-technical, see Doerfler and Bullo (2014). In the spirit of the statistical physics inspired models discussed earlier, such as Ising and spin glasses, the Kuramoto model on a complete graph belongs to the universality class of the XY ferromagnetic model (van Hemmen and Wreszinski 1993).

[4] This may be different if deliberate planning may reach completion before initiating execution (in other words, no concurrent activity is conducted while planning is ongoing). There, either the plan is then passed to the operators for execution – which may lead to its own confusions as the assumptions in the plan may not be readily 'owned' by those executing. Alternately, the planning team may become the operators, tasked with managing the operation as it goes live. This may create other problems in the planning function as it creates holes that may not be easily filled if other operations are not completed at the time of execution.

analysed. This nesting of the decision loops of military headquarters staff is not explicit in the doctrines publicly available but may be inferred. For example, the Joint Operation Planning and Execution System (JOPES) of the Joint Staff Officers Guide (Joint Forces Staff College 2000) is depicted as a two layered diagram showing the deliberate planning process above and the crisis action planning sequence below, both as linear decision sequences, with feeds between them. In the text, both the cyclic nature of these processes and the different speeds are emphasised. A similar, indeed more detailed, depiction is in NATO doctrine in the NATO Crisis Response Planning Process (Allied Joint Publication-5 2013), with three layers 'Strategic', 'Operational' and 'Tactical' and many feeds across the three. Practitioners of these, when questioned how these can be made to work, will often gesture with two hands rotating, one above and slow, the other below and fast. This is the essence of the nested loop mechanism.

I emphasise the purpose of this paper is not to evaluate any existing C2 structure, but to demonstrate how a modelling approach focused on the nested loop mechanism may be used for evaluation of any headquarters given its structure, process and the heterogeneity of its individual staff, and to identify friction points that may be addressed through restructure. For the same reason, despite the heterogeneity, I do not average over multiple instances of the random variables implicit in the model but use one fixed instance in order to illustrate the perspective of one group of individuals in the system. At the end of the day, individuals experience only one instance of their organisation.

The paper is structured as follows. First I outline the mathematical formulation of the nested loop synchronisation model in general. I then instantiate the model with a caricature of a headquarters, considering one version more hierarchical and another more networked. I then numerically solve the equations for this model instantiation across a range of coupling strengths, identifying the `stress points' in the structure. I will then discuss the implications of the behaviours of the model for human agents in a real headquarters organised in these ways. I will then conclude and outline future work.

**Nested loop synchronisation**

Consider two groups of agents, fast and slow, distinguished by the speed at which they naturally would seek to complete a cycle. These agents are connected across a network described by an adjacency matrix

$$A_{ij} = \begin{cases} 1, \text{if } i \text{ is connected to } j \\ 0, \text{otherwise}, \end{cases}$$

with *i,j* labelling the nodes of the network. This network connects both slow with slow agents, and fast with fast, but also slow with fast. For simplicity I assume the network is undirected, though the extension to a directed graph is straightforward (Kalloniatis 2016). There are then the following sets of variables:

$$\theta_i^{fast}(t), \theta_i^{slow}(t); \omega_i^{fast}, \omega_i^{fast}.$$

The $\theta$ variables are thus functions of time *t* and carry a subscript *i* indicating their position in a network. The $\omega$ are termed native frequencies and represent the speed with which an agent progresses through a cycle when left in isolation. Clearly, from the notation

$$\omega_i^{fast} > \omega_i^{fast}.$$

The frequencies will be drawn from random distributions $\mathcal{D}$ satisfying this condition. With the minimal assumption that the distributions are characterised by a well-defined mean, I draw the frequencies thus:

$$\omega_i^{fast} \in \mathcal{D}[\overline{\varpi}^{fast}], \omega_i^{slow} \in \mathcal{D}[\overline{\varpi}^{slow}]$$

The essence of the nesting is that on average a certain number *n* of fast cycles complete within the course of a single slow cycle:

$$\overline{\varpi}^{fast} = n\overline{\varpi}^{slow}.$$

For simplicity – the aim is to show the viability of such a model – I assume *n*=2 and that the distributions are uniform, of some width about the mean. Thus, on average, fast agents cycle twice as fast as slow agents, or fast agents complete two cycles for every single slow agent cycle.

The nature of the connection across any link of the network may also be characterised by the strength of *coupling*, essentially the speed of responsiveness of one agent to a change in the state $\theta$ of the connected agent. This may differ across every link of the network, but for simplicity again I will assume it to be the same – a single positive real valued constant, $\sigma$.

The model for the dynamics, using the Kuramoto model as a paradigm, is (Terada and Aoyagi 2016):

$$\begin{aligned}\dot{\theta}_i^{fast} &= \omega_i^{fast} + \sigma \sum_{j \in fast} A_{ij} \sin(\theta_j^{fast} - \theta_i^{fast}) + \sigma \sum_{j \in slow} A_{ij} \sin(n\theta_j^{slow} - \theta_i^{fast}) \\ \dot{\theta}_i^{slow} &= \omega_i^{slow} + \sigma \sum_{j \in fast} A_{ij} \sin(\theta_j^{slow} - \theta_i^{slow}) + \sigma \sum_{j \in slow} A_{ij} \sin(\theta_j^{fast} - n\theta_i^{slow}).\end{aligned} \quad (1)$$

The essence of this model is that connected agents seek to locally synchronise to the same phase within a cycle. The coupling strength represents the effort required of agents to respond to changes in state of those to whom they are connected. The sine function ensures that when similar agents (slow-slow, or fast-fast) are close to each other modulo $2\pi$, the one ahead slows down slightly and the one behind speeds up slightly so as to match phases; when they are $\pi$ apart, then the speed-up/slow-downs are reversed. The second set of terms adjust for the property that fast phases need to match twice the slow phases. The model may be generalised by making the couplings for 'fast' and 'slow' agents different; for model parsimony I do not use this. Note that the model may further be generalised to the Kuramoto-Sakaguchi case (Sakaguchi and Kuramoto 1986) where sin(x) is replaced by sin(x-$\alpha$), with $\alpha$ some lag. I do not exploit this property here, though its role in headquarters is also quite natural. The model collectively then represents fast agents synchronising within their fast loop, slow agents within theirs, and slow loops nesting a certain number (n=2) fast loops.

**Measuring performance and detecting critical points**

In the copious research on the Kuramoto model (Doerfler and Bullo 2014) there is a quite established method for determining the value of coupling at which a fundamental phase change occurs in order to distinguish between `disordered' and `ordered' states. This is via the so-called Kuramoto order parameter (Kuramoto 1984)

$$r = \frac{1}{N}\left|\sum_j e^{i\theta_j}\right|. \quad (2)$$

Values of *r* close to one correspond to highly ordered, or synchronised states, and those close to zero disordered or incoherent states.

In the case of nested loops, because the design of the model is to allow synchronisation within the separate fast/slow elements, this quantity is computed for the two separate organisational units:

$$r_{fast} = \frac{1}{N_{fast}} \left| \sum_j e^{i\theta_j^{fast}} \right|, r_{slow} = \frac{1}{N_{slow}} \left| \sum_j e^{ni\theta_j^{slow}} \right|. \qquad (3)$$

Typically here, a time-average (after throwing away an initial transient) of these quantities is considered for each value of coupling.

For large *N* `complex' networks, such as complete, random, scale-free or small-world graphs (Arenas et al. 2008) the transition from zero to one of the average *r* as coupling is varied is *sharp*. In these large *N* studies, detecting where the averaged *r* first deviates from zero (because it shoots up to value one rapidly with a small subsequent change in coupling) becomes the test of a phase transition, and is called the *critical point*. This is why critical onset of synchronization in this case is called `spontaneous' - it appears as if from nowhere.

However, for any finite system, the transition in *r* from low to high values is more gradual, and testing for critical coupling based on deviation of the average *r* from zero gives quite small values of coupling, consisting with a mostly chaotic system. In my research, drawing upon work on other models of phase transitions (Wang, Lizier and Prokopenko 2011) I have identified an alternative method of finding a critical coupling using what is known as the *Fisher information,* denoted $\mathcal{F}$. This quantity determines the sensitivity of the probability density *P* for a random variable *X* to changes in some control parameter (anticipating how I will use it for the Kuramoto model) $\sigma$. It is defined using the expected value **E** of the rate of change of the logarithm of the probability density for fixed $\sigma$. Mathematically, it can be defined, and approximated using the Newton method for discrete increments of $\sigma$, by the following series of equations:

$$\begin{aligned} \mathcal{F} &\equiv \mathbf{E}\left[\left(\tfrac{\partial}{\partial\sigma}\log P(X;\sigma)\right)^2 \middle| \sigma\right] \\ &= \prod_i \int dX_i P(X_i;\sigma)\left(\tfrac{\partial}{\partial\sigma}\log P(X_i;\sigma)\right)^2 \\ &= \prod_i \int dX_i \frac{1}{P(X_i;\sigma)}\left(\tfrac{\partial}{\partial\sigma}P(X_i;\sigma)\right)^2 \\ &\approx \tfrac{1}{\Delta\sigma^2}\prod_i \int dX_i \left[P(X_i;\sigma) + \frac{P(X_i;\sigma+\Delta\sigma)^2}{P(X_i;\sigma)} - 2P(X_i;\sigma+\Delta\sigma)\right] \end{aligned} \qquad . \quad (4)$$

This quantity may assume arbitrarily large values. Because I wish to compare it to the Kuramoto order parameter which is bounded by one, I will work with the normalised quantity:

$$\mathcal{F}_n \equiv \frac{\mathcal{F}}{\mathcal{F}_{\max}} \leq 1.$$

As the phase $\theta(t)$ may grow with time, it is best to work with a compact variable. Hence I choose:

$$X_i(t) \equiv \arcsin[\sin(\theta_i(t))] . \qquad (5)$$

Thus, the normalised Fisher information may be determined for Kuramoto-like systems by numerically solving it for the phases $\theta(t)$ over a long period of time, determining then *X* for each node, and then forming normalised histograms of the values to give the density *P*. Performing this at discrete values of coupling $\sigma$, the Fisher information is then computed from the last line of Eq.(4). The key use for the Fisher information here is that it is observed to show a peak at the value of the control parameter giving a phase transition, or triggering the shift from order to chaos, in models such as random Boolean networks (Wang, Lizier and Prokopenko 2011). A similar peak is seen for the Kuramoto model for very large networks; however its position in coupling value and sharpness varies in character depending on the network topology (Kalloniatis, Zuparic and Prokopenko 2017).

**Mapping Kuramoto to C2**

To reiterate the mapping first proposed in (Kalloniatis 2008), the phase $\theta(t)$ represents the point in a continuous decision (or OODA) cycle of an agent at some time *t*. The network represents the C2 structure itself, which may be the formal lines of authority, the information flows or even informal relationships. The frequency $\omega$ is how many decision cycles per unit time can be achieved by agent $i$ . This is chosen from a random distribution, representing the underlying heterogeneity between individual decision makers in the C2 system. The novel aspect in this paper is that there may be two (or more) disjoint distributions from which the native frequencies are selected. The coupling $\sigma$ is a measure of the strength of interactivity between C2 nodes. There is much scholarship around coupling in organisational theory and C2, including Weick (1976), Perrow (1984), Beekun and Ginn (2001a,b), Lloyd, Markham and Dodd (2006), and Stanton (2006). Weick, referring to 'loose coupling' (in contrast to 'tight') as advantageous to an organisation, lists some 15 notions of coupling that may be aggregated into two exclusive notions: coupling as connectivity, and coupling as intensity. Coupling is therefore *measurable*, though most attempts have devolved to the former using network metrics (Beekun and Ginn 2001a,b) rather than strength, intensity or frequency of interaction on the network links. In my approach, coupling may empirically refer to the speed of change in decision state by one node in response to a change of decision state by a connected partner or adversary. The $2\pi$-periodicity of the sine function is appropriate in that it locally synchronises decision cycles within the 'current phase'.

For agents in a C2 system, given that coupling implies *input* effort and degree of synchronisation is *output* performance, a critical point in the transition means there is a region where a slight increase of effort leads to a dramatic increase of output. This is why a critical point is of such interest for human organisations concerned with optimising output for input effort. But, as real C2 structures are finite, such a spontaneous appearance of coherent behaviour is not guaranteed – improved synchronisation output will generically be proportionate to the degree of coupling effort by C2 actors.

The Kuramoto model is ultimately a model quantifying *self-synchronisation* on networks. Certainly in the NCW literature, such as Alberts and Hayes (2007) and references therein, the intended self-synchronisation through the NCW tenets describes *activity* in the external environment. I propose that the precursor to this is synchronisation of *decision cycles* and therein mapping the phase of the Kuramoto model to the decision cycle; another implementation of the Kuramoto model is possible at the level of activity and is that used in (Dekker 2007). These two options are not very far apart: a decision cycle in a context such as a headquarters will very often leave a trail of external artefacts (published or draft documents, emails, chat or verbal communication) that indicate the stage of OODA of a unit or individual; these artefacts are thus points of reference for another in the same organisation in synchronising their cycle. In other words, even the cognitive stages of Observe-

Orient-Decide involve some form of social enterprise, when one steps beyond Boyd's original application to the isolated fighter pilot alone in the cockpit.

**Two alternate headquarters structures**

To illustrate the application of the model and these techniques to C2 I apply them to two headquarters models of 48 staff officers, labelled HQ1 and HQ2. I include equal numbers of planners and operators, 24 each. I assume both headquarters to be structured along Common Joint Staff System (CJSS) lines: J1=Personnel; J2=Intelligence; J3=Operations; J4=Logistics; J5=Plans; J6=Information Systems (IS). I will model up to the Directorate level to capture the planning-execution distinction. Thus the key units are (using names often used in the Australian setting): J13=Personnel Operations; J23=Intelligence Operations; J33=Current Operations; J43=Logistics Operations; J63=IS Operations; J15=Personnel Plans; J25=Intelligence Plans; J45=Logistics Plans; J55= Strategic Plans; J63=IS Plans. The missing elements from these lists are the cross-over points: J35=Operations Plans; and J53=Plans Operations.

To elaborate further on this first structure, the J35 integrates the Operations focused staff in the Ja3 (a=1,2,3,4,6) directorates; the J53 integrates the Planning focused staff Jb5 (b=1,2,4,5,6). Because J33 and J55 are somewhat exclusively focused on the very short or long term, J33 purely reactive, J55 purely deliberative, they will be the least connected. Put another way, the function of the J33 is to provide maximal SA from the tactical environment of the Area of Operations as in a Situations Centre or Joint Operations Centre (Allied Joint Planning-5 2013). The function of the J55 is to provide intent from the strategic leadership. Either way, drawing the J33 or J55 into the detailed aspects of a specialist function (logistics or intelligence) may be deemed to be undesirable. Of course, alternative models may be advisable – but this reflects arrangements the author has often encountered in Australian command units.

At the lowest level, each directorate consists of four individuals that may be considered a `team'. Connecting these teams, I consider the two structures HQ1 and HQ2, both essentially hierarchical but the second with higher connectivity. To explain the differences between the two structures I focus on logistics specialists, planners J45 and operators J43. In the first structure, individual functional specialist planning teams, such as the J45, are a complete graph of four, with one team leader whose purpose is to link, on the one hand to the lead planner in the J53, and on the other to link into the lead functional specialist for the operation, the J43. In this way, relevant logistics information from the operation may be fed into the logistics planning. Similar considerations apply then to the J15, J25, and J65. And, *mutatis mutandis*: the J43 leader links to the operations leader in J35, as well as to (as mentioned) the lead logistics planner in J45. The exceptions to this pattern are the J55 in strategic planning who only link to the J53, and the operations monitoring team the J33 who link to the J35 (also previously discussed).

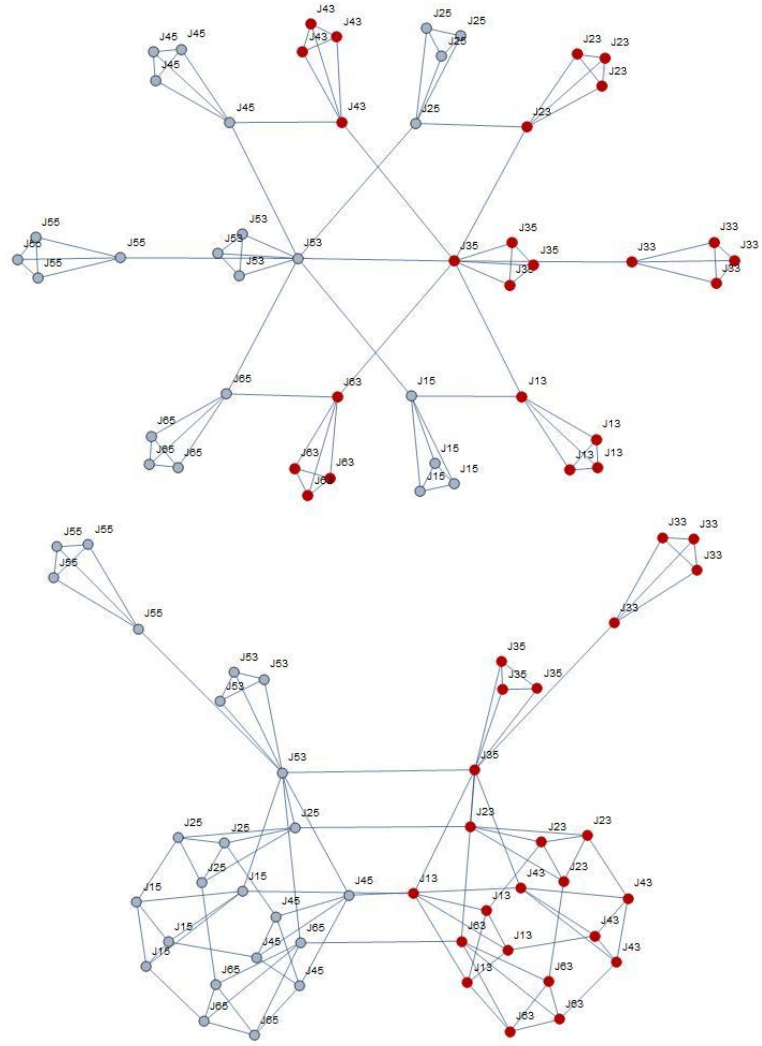


*Figure 1 Two headquarters structures, HQ1 (top) and HQ2 (bottom). CJSS numbers are applied down to the team level, with individual team members not distinguished here.*

In the second structure, these relationships are augmented with liaison team member links in each functional specialist team, not the leader, who links to a corresponding liaison member in another functional specialisation; every team member becomes a liaison with a different specialist area so as to spread the burden (albeit imperfectly, as will be seen shortly). In HQ2, the J55 and J33 are kept loosely connected (in terms of degree) to the structure as in HQ1.

In Figure 1, I show the two headquarters structures with CJSS numbers applied; here I do not distinguish individuals within teams though, of course, each node has its own index ($i$=1,…,48) enabling such a distinction to be made when solving the equations. The links in the graph in such a simplistic depiction of C2 may be seen as more than just the formal lines of authority since even HQ1 is more than a pure tree-hierarchy; rather these are lines of information flow such that agents are able to adjust their individual decision-loop in light of interactions with others. In this respect, the interaction of different organisational process is captured: the nesting of a fast execution cycle inside a slower planning loop. Of course, the intent of HQ2 is to spread more load onto team members so that there is less of a disparity of node degree across the network. As seen in Figure 2 this is not perfectly achieved: the lead operator in J35 and lead planner in J53 both have degree 9, compared to some team members with degree 3. This does, however, reflect the disparity in workload in managing across multiple areas up the military rank hierarchy. I will return to this in the conclusions.

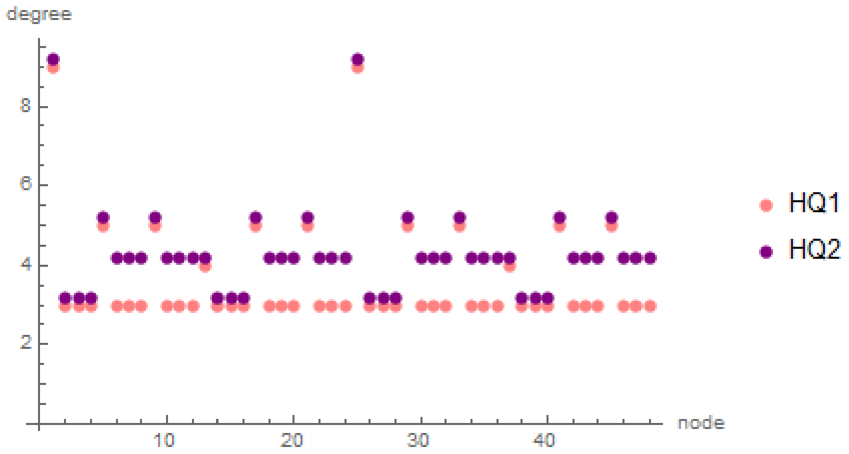


*Figure 2 Degree distributions for the two headquarters, HQ1 and HQ2, where nodes 1-24 are the operators, or 'fast' agents, and nodes 25-48 are planners, or 'slow' agents.*

**Numerical calculations**

In the following I present results obtained by numerically solving the equations for each headquarters structure up to time t=2000 (when the system synchronises this suffices to have the coherent dynamics overwhelm the transients), coupling steps of 1/200 up to 1.75. The Mathematica software package is used. One instance of native frequencies and initial conditions is used – the same for both headquarters structures. The native frequencies are selected from uniform distributions $\mathscr{U}(0.25,0.5)$ for slow and $\mathscr{U}(0.5,0.75)$ for fast agents. This choice captures one element of reality – no individual is perfectly 'brilliant' ($\omega$=1) or 'stupid' ($\omega$=0). The overall scale of the maximum frequency is inconsequential – it may be absorbed into the coupling strength. In Figure 3, I show a histogram of the frequencies selected in this study.

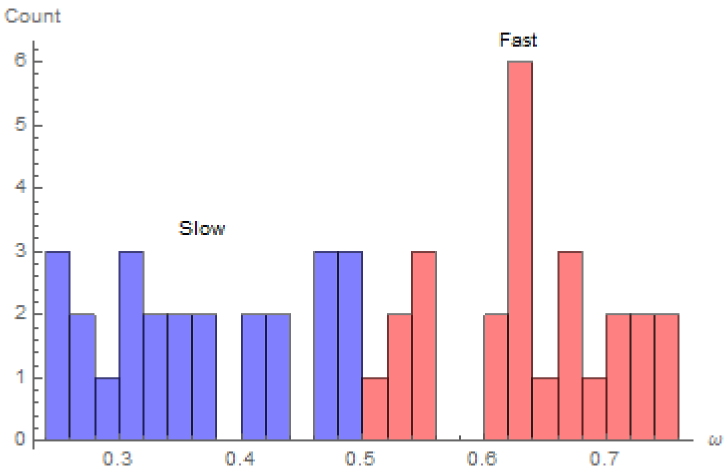


*Figure 3 Histogram of frequency choices used in numerical calculations*

It is important to bear in mind in the following that once a transient in the dynamics is overcome, the behaviour of individual agents is a consequence both of their structural connectivity and their native frequency; because I do not average over frequency instances, topology alone cannot explain certain results though it does explain most. In computing the Fisher information, the time-series data for each *X* is aggregated in 20 bins through interval ($-\pi/2,\pi/2$). In discussing the numerical results I refer to 'fast' and 'slow' agents, but in the subsequent section I will contextualise these results in the headquarters construct.

I first show the results for the order parameters and the normalised Fisher information for various sets of agents in Figure 4.

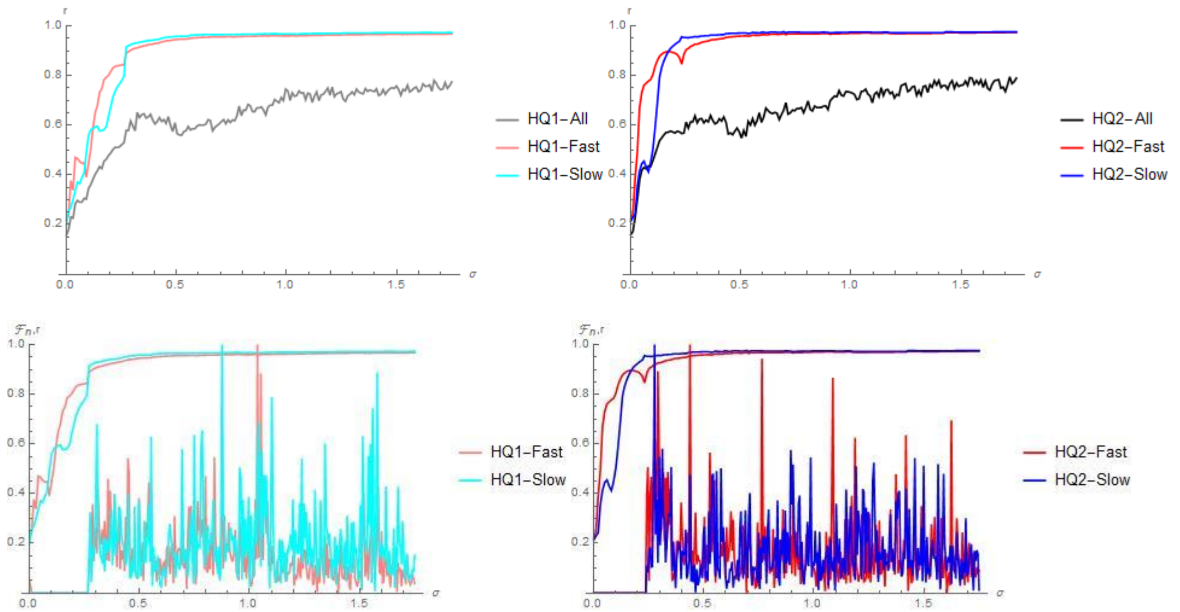


*Figure 4 Order parameters and normalised Fisher information: on the left for HQ1 and on the right for HQ2. The top row only shows the order parameters r for fast (pink/red), slow (cyan, blue) and all (grey/black) agents. The bottom row shows the order parameters compared to the normalised Fisher information, shown in the zig-zag lines, for fast/slow agents.*

Numerous observations may be made here. Focusing only on the order parameters in the top row, it is clear that whereas both fast and slow agents reach near perfect synchronisation ($r\approx1$) as coupling is increased, the total order parameter shows quite moderate synchronisation ($r\approx0.7$) even at very high values of $\sigma$. This is as to be expected given the design of the model: the two sets of agents are not constrained to be perfectly synchronised between each other. Both headquarters designs achieve their high levels of synchrony at approximately $\sigma\approx0.2$-$0.3$, to be quantified more precisely soon. Closer inspection shows that HQ2, the more connected structure, achieves values of $r\approx1$ at lower coupling. However there is a wrinkle visible in the right hand plots: just as the slow agents (blue curve) achieve their highest level of synchrony, the fast agents (red curve) undergo a slight drop in their levels; the improvement in synchrony of slow agents comes at the cost of synchrony of fast agents because of the higher connectivity. Either way, it is difficult by visual inspection of the top row plots to fix a discrete value of coupling that may be deemed `critical'.

The bottom row of Figure 4 addresses this, showing that the normalised Fisher information definitively jumps from near zero values to high values at a single value of coupling: for HQ1 this is $\sigma=0.275$ and for HQ2 this is $\sigma=0.235$. Notably, compared to theoretical complex graphs (such as `scale-free' and `small-world') neither of these headquarters structures shows a clean dominant peak in the Fisher information; there is no ‘phase transition’ as such. Nevertheless, a qualitative change in behaviour of the Fisher information occurs as synchrony is approached. For simplicity, I will refer to this value as the critical point.

I now zoom into the individual phases $\theta$ at these two critical values of coupling for the respective headquarters structures (in fact, just below the point the Fisher information jumps, since a difference of coupling values is used to compute the discretised derivative in the Fisher information). I provide a view of this where I subtract out the mean behaviour of the fast, respectively slow, agents, namely I plot

$$\theta_i^{fast}(t) - \overline{\omega}^{fast} t, \theta_i^{slow}(t) - \overline{\omega}^{slow} t \ .$$

Thus, in Figure 5 the results for the two headquarters are shown left and right, with fast top and slow below and HQ1 left, HQ2 right. In both cases, it is clear that though the synchronisation overall is quite good (trajectories are `bunched’ together quite well) but there are small discrepancies. Firstly, the agents have not in fact synchronised to the mean of their respective frequency – fast agents are slightly faster (trajectories move upwards) than the mean fast frequency, and slow agents are slightly slower (trajectories move downwards) than their mean. On the one hand, this is an effect of finite sampling of the distributions from which the frequencies are drawn – but of course for any real group of individuals, they will only ever have opportunity to sample once. This is the main reason I focus on one instance. Secondly, I observe that in HQ1, for all the bunching, there is still a degree of `looseness’ in the pack: some agents in a given group (fast or slow) fluctuate earlier in response to a change elsewhere, others later. Indeed, by comparing fast and slow for HQ1 it is clear that the fluctuations are due to mutual interactions between the fast and slow groups. In HQ1, the largest fluctuation is seen in the slow agents where at t=1800 many drop in relation to the mean by an amount of approximately 6 – this is nothing other than a jump of $2\pi$.

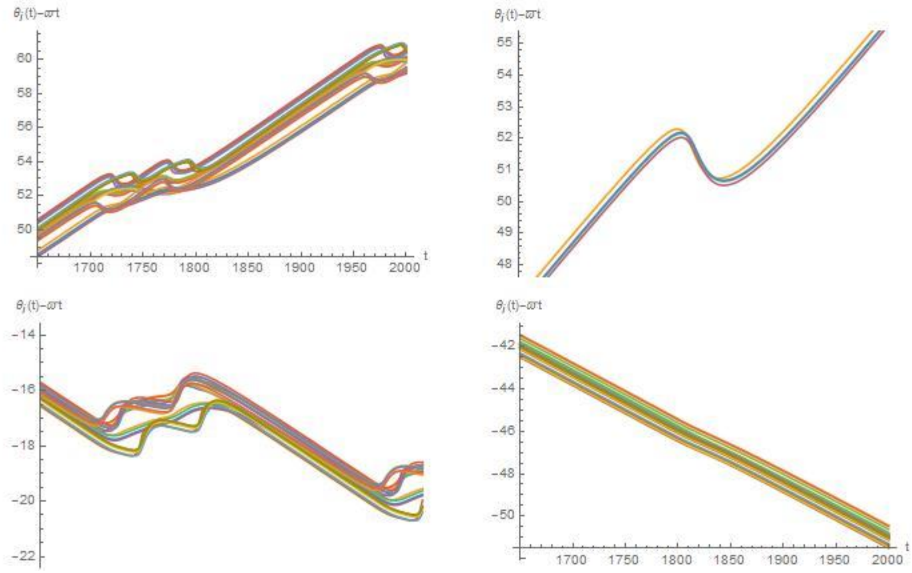

*Figure 5 Individual phases with the collective behaviour subtracted out, for HQ1 left, HQ2 right, fast agents top and slow agents bottom.*

For HQ2, phases are more tightly grouped, however a very distinct fluctuation is generated in the fast group (top right) that barely registers in the lower group. This occurs periodically through the dynamics; it is the reason why there is a dip in the order parameter for fast agents in Figure 4. Thus, the connectivity overall leads to better synchronisation but with an occasionally requirement for fast agents to `correct' their state given their links to the slow agents through the J35-J53 nexus.

It is clear in both cases that some agents undergo larger fluctuations, and are further from synchrony, than others. So, finally, I select out individual trajectories based on the scale of the fluctuations over a finite time window. These are shown in Figure 6, where I plot as a function of time the magnitude of phase differences between key pairs of nodes. The pair that show the greatest fluctuations I indicate in blue. For comparison, I plot some showing smaller fluctuations either on the same graph or in insets a number of others.

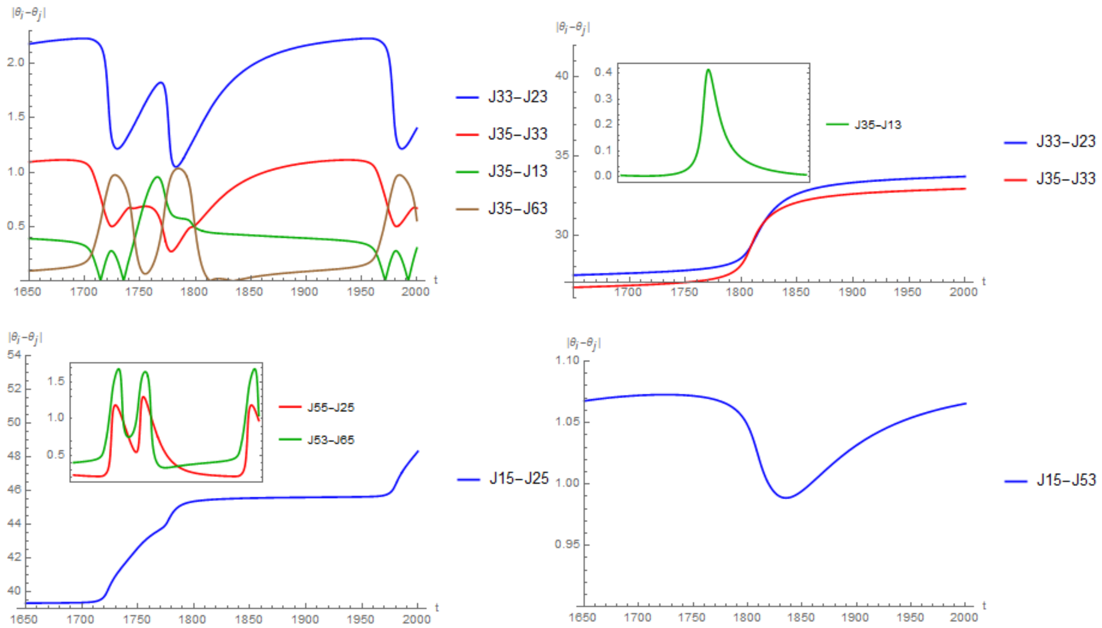


*Figure 6 Phase differences across adjacent pairs as labelled, HQ1 left, HQ2 right, fast agents top and slow agents bottom; the result for the largest fluctuations in the set are plotted in blue.*

Thus, for HQ1 (left hand plots) the largest fluctuations between fast agents (top) occur between the J33 and the J23 who never quite achieve perfect synchrony: observe that the blue curve in the left hand top plot of Figure 6 never drops down to zero. Similar patterns are seen in other pairs of units such as the J35 and J33 (red curve). Observe also how the pair J35-J13 undergo an additional, though smaller, fluctuation in the period 1700<$t$<1750. By showing these other pairs, it becomes clear that many achieve at discrete points in time perfect synchronisation – but this is not sustainable over all times. Contrastingly, as mentioned, J33-J23 never achieve this state (nor do J35-J33 – red curve – though they get closer). The situation for the slow agents is worse, with pairs such as J15-J25 undergoing an entire phase jump; observe that the fluctuation in the blue curve in the lower left hand plot is, again, by an amount of 2$\pi$ (the blue curve is somewhat deceptive with its flattening at a new value different from that before the rise: the agents are at the same phase value; however the important aspect is the size of the fluctuation being the amount 2$\pi$). Indeed, the shift in this phase difference occurs at the same time (at $t$=1720) as the corresponding fluctuations amongst the fast agents. This occurs over the entire dynamic, and for many other pairs as was shown in Figure 5. The inset plot there (left, lower plot) shows that some pairs are better synchronised, for example J55-J25, with a fluctuation of approximately 1.5.

Examining the right-hand plots of Figure 6, for HQ2 the largest fluctuation between a pair of agents in the jump mentioned earlier for fast agents is that for J33-J23. This was the same pair that suffered the largest fluctuation for fast agents in HQ1. In fact, in HQ2 it is now worse: they undergo an entire phase shift of $2\pi$. The situation is only marginally better for J35-J33 and a number of others (not plotted). This is the cause of the drop in the order parameter for fast agents in HQ2 seen at the critical point in Figure 1. Contrastingly, for a pair in HQ1 that underwent a large fluctuation comparable to that of J35-J33, namely J35-J13, in HQ2 their phase differences only `wobbles' by approximately 0.4. Amongst pairs of slow agents in HQ2, fluctuations are predictably (from Figure 4 and Figure 5) small; I only show one example between J15-J53 where the magnitude of the wobble is 0.07. Clearly, the structure in HQ2 for slow agents is more effective in enabling them to achieve high levels of synchronisation at a point where their fast counterparts are still 'struggling'. Indeed, the slow agents' success is largely (given the cross-connectivity) at the expense of the fast agents.

This analysis demonstrates that both the collective performance of the structures may be analysed, as well as identification of key points of change in the dynamics, and finally identification of individual agents participating in the most severe of the underlying fluctuations.

**Discussion: Implications for headquarters organisation**

What does this all mean for the respective headquarters organisations? An initial aspect to this question is why I have adopted the point of analysis as the critical coupling. As alluded, coupling in the organisational context is an effortful activity (Weick 1976) and, even seeing the aim of a distributed group of staff officers as achieving synchronisation, when caught between fulfilling one's individual task and that for the distributed whole, there is a principle of minimum effort to get the job done that would apply. Therefore, it is reasonable to assume that an organisation seeking to achieve self-synchronisation will attain at the lowest possible coupling that still achieves the collective purpose – that is the critical coupling.

One overriding observation from the above analysis is the persistence, across both organisational structures at their respective critical couplings, of large fluctuations between the J33 and J23 directorates. This is clearly a consequence of the poorer linking of the J33 'situation centre' into the wider operations management arrangements, including (though to a slightly lesser extent) the operations leaders in J35. Though topologically, one might apply the same logic to the J55 in the planning arrangements, it is clear the fast tempo makes this more problematic for the operators. With the importance of the operations-intelligence relationship for overall completeness of situational awareness – in other words, effective integration of the 'blue' and 'red' pictures – this may be seen as a significant flaw in the C2 structure.

Next, given the degree of connectedness of planners and operators across the system, improvement in levels of synchrony for one comes at the expense of it for the other. In HQ1 at its critical coupling, the operators achieve overall higher levels of synchrony – albeit subject to offsets (between J33-J23) and fluctuations (the rest) – than the planners; but, as a consequence of their mutual interactions, the point at which the operators achieve closer synchronisation is the point at which the planners are thrown off their cycle. The reverse occurs in the more connected headquarters HQ2: planners are better synchronised (almost perfectly), but the pull of some operators into planners' considerations has the effect of throwing the participants in the faster cycle out.

The fluctuations between agents may be seen as a proxy for cognitive (because I am modelling distributed cognition) 'stress': the effort that a planner or operator might otherwise put into their individual objective (for example, developing the logistics appreciation in J45 for the operational plan) is diverted into adjusting with respect to a peer elsewhere in the headquarters. Thus, large (in magnitude) and numerous fluctuations correlate with more stress on an individual staff officer in the organisation. In this respect, the impact of HQ1 for the operators is quite significant with 2-3 fluctuations in a cycle across most of the directorates.

The two structures, HQ1 and HQ2, highlight the conflicting demands between tempo and connectivity. Is more connection universally good for a headquarters? It depends on the tempo: for slower, more deliberate planners the answer is 'yes', whereas for the faster, reactive operators it is 'no'. Of course, in all cases better synchronisation may be achieved with higher coupling strength which in turn, as mentioned, invokes a cost in effort. To that degree, using different coupling strengths for planners and operators only hides the issue inside individual branches and directorates. With the philosophy that an organisation represents arrangements that best balance the needs of the whole with the limitations of the individual, the aim would be to find network structures that both smooth out the degree distribution across nodes while allowing for heterogeneity in connectivity to enable the balance between planners and operators.

**Conclusions and future work**

I have presented an approach to modelling both planners and operators in a headquarters C2 structure in the one representation. This exploits the idea of nested cycles in models of synchronisation on networks in the physics literature. The model either draws upon, or enables analysis with, many of the tools of modern statistical physics, such as networks, phase transitions and chaos theory. However, though abstracting in its own way, the Kuramoto-based C2 model is singular in building in the cognitive action-perception cycle that pervades many qualitative and quantitative human factors approaches.

By presenting two, somewhat caricatured, examples of headquarters structures up to the directorate level, I have instantiated the model and by numerical solution shown how the collective behaviour may be analysed and how individual behaviours may be extracted and understood. The aim of this paper was not to validate the model as such – this was pursued for the operations-focus version of the model in (Kalloniatis 2016). Nevertheless, even with the caricatured networks used here, a `face-validation' (Sargent 1984) was possible: the behaviours associated with less connected units such as the J33 and J55 here may be confirmed against the network diagrams and domain knowledge of military headquarters, as may the behaviour that the more connected HQ2 synchronises at lower coupling (with less effort) than HQ1. What cannot be determined by visual inspection are the second-order effects of degrees of synchronisation within sub-sections of the organisation, the trade-offs between time-scales and connectivity and the degrees of fluctuations between connected nodes in the C2 structure. This has been achieved here. Moreover, static network diagrams do not readily provide a test of whether a critical point may be reached: one may subjectively judge a particular C2 network as `complex', but the dynamical model enables a clean test of whether a threshold for amplified performance is attainable for a given organisational structure. Many of the other statistical physics inspired approaches to C2, mentioned in the introduction, provide for similar tests – mostly with more computational effort. While this test of criticality is intrinsic to the present Kuramoto-based model, a consistent pattern across a range of such models would provide for robustness in the expectation that a threshold is attainable for the real human organisation.

Though, as said, the purpose in this paper was not to validate the model in detail there is a subtlety that bears mention in the context of critical behaviour. Typically, in computational modelling for operations research purposes sensitivity analysis is required around the regime where recommendations for changes are derived; behaviours from a model that are sensitive to changes in uncertain parameters do not provide for reliable predictions. Of course a critical system is precisely one where such sensitivity occurs *in reality*, usually in the context of accidents but here in improved performance. To the degree that one seeks self-synchronisation to be a spontaneous phenomenon in a C2 system, such sensitivity is desirable – and has been explored here using the Fisher information. Thus the true sensitivity test for prediction *of critical behaviour in a C2 system* is its robustness across a variety of models.

Through the paper I have avoided saying that the behaviour of one agent, or staff member, 'causes' the behaviour of another. The model elegantly encapsulates the point that in a complex system cause-and-effect may not be readily disentangled. Rather, all agents mutually influence each other through the micro-adjustments that self-synchronisation relies upon. By contrasting two models of quite different connectivity the trade-offs between enhancing the performance of planners and operators have been highlighted. Of course, neither of the headquarters structures used here represent 'real' organisations which are indeed more 'complex'.

The current model may be further enhanced by combining with elements I have modelled in previous work. For example, it is straightforward to include the engagement of nodes with an environment external to them (the tactical level for J33, the strategic level for J55, or even direct interaction with adversaries or competitors), as in (Kalloniatis 2012). This may also address the imbalance in degrees across both networks, perhaps unfairly representing the load of a manager (in some cases degree 9) against that of a `worker bee' (degree 3); the latter would argue that managing their own work *in depth* matches the manager's work *in breadth*. This can be achieved by adapting the so-called 'Blue-v-Red' model (Kalloniatis 2012) to this context, so that staff engaged directly with the external environment link to additional nodes with their own native frequencies. In contrast to the interactions modelled here, these agents will seek a phase lag ahead of the external agent ("get inside the adversary OODA loop").

This nested loop approach may be enriched further. Stochasticity may be introduced to represent both the fog-and-friction of the operational environment (Kalloniatis and Zuparic 2014) and the property that human decision-making does not smoothly proceed through its cycle (OODA, Perception-Action or other), but exhibits both jumping through intuition, or recognition priming (Klein 1998) and halting (through indecision), as used in (Kalloniatis 2016). I have also simplified in this paper by allowing connected agents in the perfect knowledge of the 'OODA' state of each other, generally not the case in a real organisation. However, I have shown elsewhere (Kalloniatis 2012) that this may be overcome by overlaying the interaction in equations such as Eq. (1) with distributions peaked in certain directions of the loop. This has the effect of modulating the strength of the interaction to coincide with points in a cognitive process when one agent may reveal to another their decision state; the system becomes more stochastic causing an overall shift up in the required critical coupling.

It is clear, then, that C2 brings together many diverse mechanisms of human individual and organisational decision making, but each of these may be modelled individually and collectively. To conclude definitively, this work completes the last piece of a large jigsaw puzzle of elements to bring the elegant Kuramoto model as a paradigm of self-synchronisation to bear upon the richness of headquarters C2. Across numerous works, I have shown glimpses of aspects of a larger model and how they reflect aspects of the reality of military headquarters life, including people and technology. All that remains now is to bring these altogether as a basis for exploring the fitness of different C2 approaches for the variety of contingencies they are intended to confront.

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